Stage M2: Model learning with rational optimization and constraints

General information

- **Candidate profile** : second/third year engineering school and/or master of science with strong skills in statistics, machine learning, optimization. Languages : Python/Matlab ; web services is a plus
- **Duration** : 4/6 months
- **Location** : Center for Visual Computing, OPIS, INRIA, CentraleSupélec, Université Paris-Saclay.
- Supervision : **Marc Castella** (marc.castella@telecom-sudparis.eu, Telecom SudParis)
- Co-supervision : Jean-Christophe Pesquet (jean-christophe.pesquet@centralesupelec.fr, OPIS, INRIA Paris Saclay, CentraleSupélec) and Laurent Duval (laurent.duval@ifpen.fr, IFP Energies nouvelles)

Subject

The context of the subject is modeling of experimental data — $\mathbf{y} = A_\theta(\mathbf{x})$ — where A_θ denotes a parametric family of functions. For a long time, polynomial and rational functions have been playing a key role in that respect (calibration, saturation, system modeling, interpolation, signal reconstruction. . .) Their use is classically confined to least-square or least-absolute value regression. However, it is highly desirable in real-world applications to incorporate:

- data-based penalties (positivity, interval bounds),
- model and noise priors, such as power laws or variance stabilizing transforms [Ans48],
- statistical priors, such as robustness or sparsity [Sob81].

Basically, from the modeling perspective, any cost function of practical interest can be approximated as accurately as desired by a polynomial. Nonetheless, optimizing multivariate rational function under polynomial constraints is a difficult problem when no convexity property holds. Recent mathematical breakthroughs have made it possible to solve problems of this kind in an exact manner by building a hierarchy of convex relaxations [Las01, CPM19].

The objective of this internship is to pursue the developments in [MCPD18, MCP18] applied so far to signal restoration. We address here the quest for best model selection in experimental data fitting. We minimize a criterion $\mathcal{J}$ composed of two terms. The first one is a fit measure between the model and recorded measurements $\mathbf{x}$ and $\mathbf{y}$. The second terms are sparsity-promoting (approximating the ℓ_0 pseudo-norm) and interval bound penalizations, weighted by positive parameters λ and μ.

$$\mathcal{J}(\mathbf{x}) = \underbrace{\|\mathbf{y} - A_\theta\,(\mathbf{x})\,\|_2^p}_{\text{data fidelity}} + \underbrace{\lambda\Psi(\theta) + \mu\Phi(A_\theta\,(\mathbf{x}))}_{\text{penalization}}\,.$$

We choose Φ, Ψ and A_θ as rational functions. The latter will be dealt with by subclasses (e.g. polynomials, homographic functions). The proposed methodology will be evaluated on the many models and experimental data available at IFP Energies nouvelles. A particular attention will be paid to the usability to data practitioners, by embedding the algorithms into an user-friendly interface, with help in choosing parameters p, λ and μ.

References

[Ans48] F. J. Anscombe. The transformation of Poisson, binomial and negative-binomial data. *Biometrika*, 35(3/4):246–254, Dec. 1948.

[CPM19] M. Castella, J.-C. Pesquet, and A. Marmin. Rational optimization for nonlinear reconstruction with approximate ℓ_0 penalization. *IEEE Trans. Signal Process.*, 67(6):1407–1417, 2019.

[Las01] Jean B. Lasserre. Global optimization with polynomials and the problem of moments. *SIAM J. Optim.*, 11(3):796–817, Jan. 2001.

[MCP18] A. Marmin, M. Castella, and J.-C. Pesquet. Sparse signal reconstruction with a sign oracle. In *Proc. Signal Processing with Adaptive Sparse Structured Representations (SPARS) workshop*, 2018.

[MCPD18] A. Marmin, M. Castella, J.-C. Pesquet, and L. Duval. Signal reconstruction from sub-sampled and nonlinearly distorted observations. In *Proc. Eur. Sig. Image Proc. Conf.*, 2018.

[Sob81] E. Sober. The principle of parsimony. *Brit. J. Phil. Sci.*, 32:145–156, 1981. Isn't it ironic, don't you think? (Sober as an author made for parsimony).